A credit card allows you to buy now and pay later, but if you do not pay off the bill within a short period, interest is added to what you owe. The rate of interest varies from one card to another, often being about 1 to 2% a month. This does not sound much, but can accumulate to a substantial amount.

This activity shows you how to work out how long it takes to pay off a credit card bill and how much extra you will pay if you decide to pay off just a small amount each month.

Information sheet

Suppose you spend £1250 on various purchases and pay using a credit card that charges interest at a rate of 1.2% per month. Assume that:

- you pay back £80 each month until the balance is less than £80 and then you make one final payment to clear the debt

- you buy nothing more with the credit card.

The amount owed at the end of each month
    = Amount owed at the beginning of the month + Interest – Amount paid

Suppose £B_n denotes the balance (i.e. the amount owed) at the end of the nth month. This is also the amount owed at the start of the $(n + 1)^{th}$ month.

The amount owed at the end of the $(n + 1)^{th}$ month is

$$B_{n+1} = 1.012B_n - 80 \quad \text{.......(1)}$$

Think about…

Can you explain this expression?

Relationships like $B_{n+1} = 1.012B_n - 80$ are called recurrence relations. Given a starting value, B_0, you can use the recurrence relation over and over again to find the subsequent values B_1, B_2, B_3, ….

In this case the 'starting value', B_0 is the initial debt of £1250.

Using this as B_0 gives
$$B_1 = 1.012 \times 1250 - 80 = 1185$$
$$B_2 = 1.012 \times 1185 - 80 = 1119.22$$
$$B_3 = 1.012 \times 1119.22 - 80 = 1052.65 \qquad \text{and so on.}$$

It may seem a tedious job to carry out a long sequence of such calculations to find out when the debt will be paid off, but it can be done quickly using the Answer key on a calculator. Spreadsheets also provide a quick method for recurrence relations if you use the 'Fill down' command to find successive values.

Try one, or both, of these methods as explained below.

Try this: Using a graphic calculator

Enter the value of B_0, in this case 1250, into your calculator.

Now enter the recurrence relation $B_{n+1} = 1.012B_n - 80$ using the Ans key as B_n (i.e. $1.012Ans - 80$)

Repeatedly press the 'equals' key to give successive terms in the sequence. Check that the values you get agree with those given in the table and complete it.

At the end of the 17th month you should find that there is only £32.28 left to pay off.

Think about…

If you pay off the balance at the end of the 17th month, how much will your purchases have cost altogether?

How much will you have paid in interest?

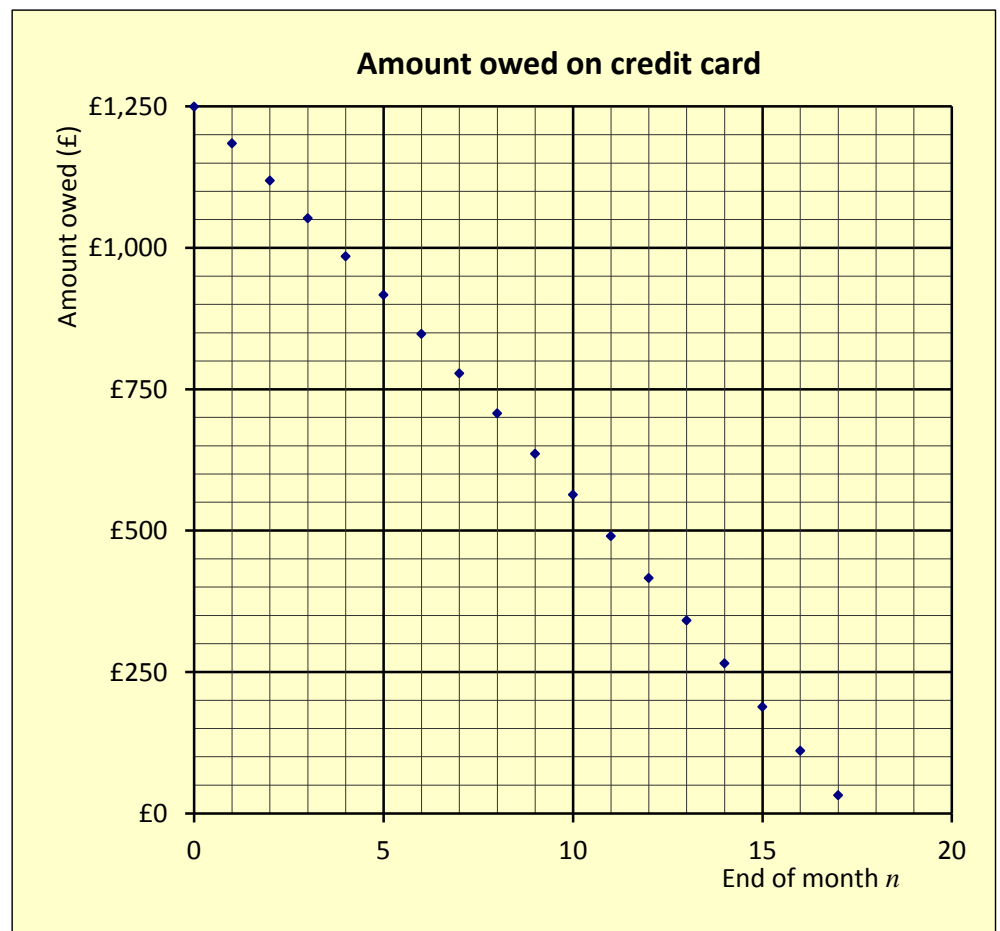Express the interest as a percentage of the original price.

End of month	Balance
0	£1,250.00
1	£1,185.00
2	£1,119.22
3	£1,052.65
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	

The way in which the debt decreases can be shown on a graph.

Complete this graph.

Think about…

Do the points lie on a straight line or a curve?

Amount owed on credit card

Try this: Using a spreadsheet

The recurrence relation that gives the amount owed at the end of each month is $B_{n+1} = 1.012B_n - 80$

The way this is entered into a spreadsheet is shown below.

Think about…

how the formulae used in cells B3, B4 and B5 follow from the recurrence relation.

Copy the contents of cells A1 to B3 onto a spreadsheet and use 'Fill down' to extend the table.

	A	B
1	**End of Month**	**Balance**
2	0	1250
3	=A2+1	=1.012*B2-80
4	=A3+1	=1.012*B3-80
5	=A4+1	=1.012*B4-80

You should find the debt reduces to £32.28 at the end of the 17th month.

Think about…

If you pay off the balance at the end of the 17th month, how much will your purchases have cost altogether?

How much will you have paid in interest?

Express the interest as a percentage of the original price.

Use columns A and B to plot a graph of the amount owed against time.

Think about…

Do the points lie on a straight line or a curve?

Extension

Imagine you are buying something expensive with a credit card. Investigate how long it would take to pay off the debt, and how this varies with different interest rates and different regular payments.
Write a summary of what you find.

Reflect on your work

If you tried both a graphic calculator and spreadsheet, which do you prefer? Why?

In practice what complications can make the working more difficult?